PARAMETER IDENTIFICATION OF AN EQUIVALENT CIRCUIT MODEL VIA SUPPORT VECTOR REGRESSION FOR FREQUENCY SELECTIVE SURFACES

Lorena Fernandez-Martinez[(1)], **E. de Lera-Acedo**[(1) (2)], **E. Garcia-Muñoz**[(1)], **A. Navia-Vazquez**[(1)], **D. Segovia-Vargas**[(1)]

(1)*Dpto. de Teoria de la Señal y Comunicaciones, Universidad Carlos III de Madrid, Avda. Universidad 30, 28911 Leganes, Spain*

(2)*Centro Astronómico de Yebes – Observatorio Astronómico Nacional 19080, Apdo. 148, Yebes, Guadalajara, Spain*

lorefer@tsc.uc3m.es, eloy@tsc.uc3m.es, legarcia@tsc.uc3m.es, navia@tsc.uc3m.es, dani@tsc.uc3m.es

ABSTRACT

A method for the analysis and design of Frequency Selective Surfaces (FSS) based on Support Vector Regression machines is developed. FSS representation with an equivalent circuit composed of lumped elements is very useful in the design stage of systems, where it is desirable to know in a quick way if a FSS is able to give a fixed frequency response. A high number of equivalent circuit elements are necessary in order to build the design model. The way to obtain these elements is often performed by an analysis procedure based on time-cost full-wave simulations.

The use of Support Vector Regression machines speeds up this procedure, making easier the analysis step. These machines are likely to bring good results also in the design step, instead of the complex equations typically used.

The method will be validated with a periodic structure of metallic ring resonators on a dielectric substrate.

1. INTRODUCTION

Support Vector Machines (SVM) for regression problems, the so-called SVR (Support Vector Regression), were proposed by Vapnik [1] in 1995. Unlike the original machine SVM for classification problems, SVR only depend on the training samples subset to be found outside a margin which is defined by the maximum deviation free parameter ε. Even though SVR machines have not shown a performance as good as the SVM ones, they constitute a powerful tool in regression problems when dealing with reduced data training sets in high dimensional spaces. This will be the scenario in which we estimate the Frequency Selective Surface circuit equivalent parameters.

Special attention have been paid to FSS in last years in a number of applications, for instance, electromagnetic-filtering devices for reflector-antenna systems, radomes, absorbers, and artificial electromagnetic-bandgap materials.

A FSS is a planar structure comprised of one or more metallic layers, each backed by a thin dielectric slab. The FSS screen has a two-dimensional periodic design described by a unit cell. When an incident wave impinges on this structure, the periodic features in the metallic sheet resonate at certain frequencies, depending upon the dielectric properties of the substrate and the geometry of the unit cell [2]-[3].

The representation of a FSS using an equivalent circuit speeds up the design stage: work in a circuital domain is faster and more efficient than work in a full-wave domain.

Our design methodology consists of obtaining a heuristic model that allows us to go from the full-wave simulation domain FSS to a circuit simulation domain. For this purpose, a SVR is used to obtain the equivalent circuit parameters in order to reduce the computational burden.

2. FSS COMPOSED OF RING RESONATORS ANALYSIS

A periodic structure of metallic ring resonators on a dielectric substrate has been used to analyze the method.

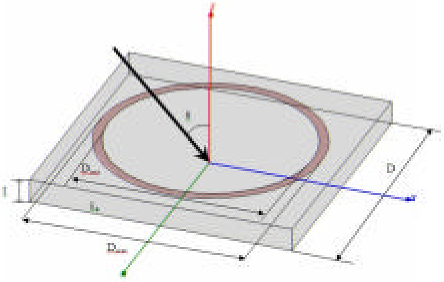

Figure 1. Geometry of unit cell

Proc. 'EuCAP 2006', Nice, France
6–10 November 2006 (ESA SP-626, October 2006)

The plane wave impinges on the structure through an angle θ. Due to the periodicity of the structure; a unit cell can be cut out and analyzed.

Fig. 1 shows the ring structure where D is cell's size, D_{ext} and D_{int} the inner and the outer diameter of the metallic ring, t is substrate thickness and ε_r is the relative permittivity.

In order to validate our software we have chosen this electrical structure because it has been widely studied [4]-[5]. Its frequency response is shown in Fig. 2 where the design parameters are in Table 1.

	FSS
t (mm)	0.64
e_r	11
$\mathbf{D_{int}}$ (mm)	5.6
$\mathbf{D_{ext}}$ (mm)	6.1
D (mm)	7.24
q	45º

Table 1. Ring parameters

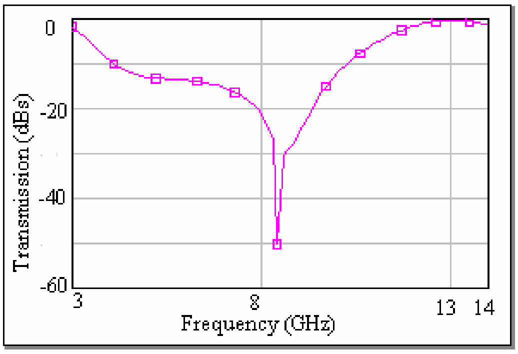

Figure 2. Transmission Coefficient calculated with the parameters shown in Table 1

Fig. 3 shows the transmission coefficient for different impigning angles.

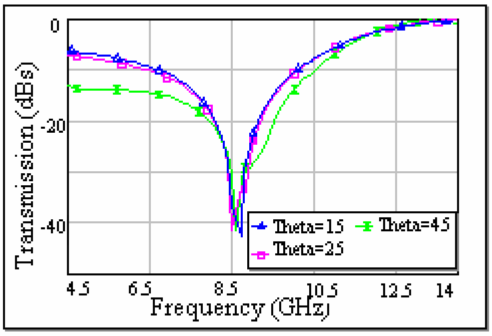

Figure 3. Variation in the incident angle

An important characteristic of this structure can be extracted from Fig. 3: it is very stable in frequency with the variation in the incident angle, up to incident angles of 50º.

3. FSS EQUIVALENT CIRCUIT MODEL

When an equivalent circuit of a FSS is not available, the method used in its design is the theory of the "trial-and-error", as stated by K. Lee in [6]. In order to avoid this tedious technique, we aim to find an equivalent circuit that represents a particular structure. For simplicity, the proposed equivalent circuit is a resonant LC circuit, whose values will be obtained as a function of the electrical and geometrical FSS parameters. Once an equivalent circuit has been obtained, it is possible to calculate the frequency response of the FSS in a faster and more efficient way, what reduces the time in the FSS design. This kind of analysis has already been carried out for other structures ([5]-[10]).

The equivalent circuit model can also be employed in the design of the multi-layered FSS, also known as Frequency Selective Volume (FSV), because a circuit model capable of predicting the behaviour of an isolated layer is very important in the design process of the FSV. First, an equivalent circuit of each layer is calculated and then the layer effect is computed by cascading the individual responses. If this approach is used, it is not necessary a specific strategy for multilayer structures, which would be more inefficient and time expensive [7].

4. RING RESONATOR EQUIVALENT CIRCUIT

Two properties define the equivalent-circuit model. On the one hand, an intense full wave domain study for the FSS under analysis clearly shows a resonant behaviour. On the other hand, the motive (metallic structure properties) and the periodicity are obtained from the visual analysis of the FSS.

4.1. Motive

The equivalent circuit is extracted by taking into account only the geometry of an isolated metallic ring. The motive is associated with two circuit elements: an inductance which is related to the metallic width, and a capacitor whose value depends on the charge storage at the metallic ring.

4.2. FSS Periodicity

The presence of the adjacent rings introduces two additional circuit elements, again an inductance and a capacitor. The inductance represents the mutual inductance between two rings and it is affected by the distance between them. The capacitor is associated with the geometry of the ring.

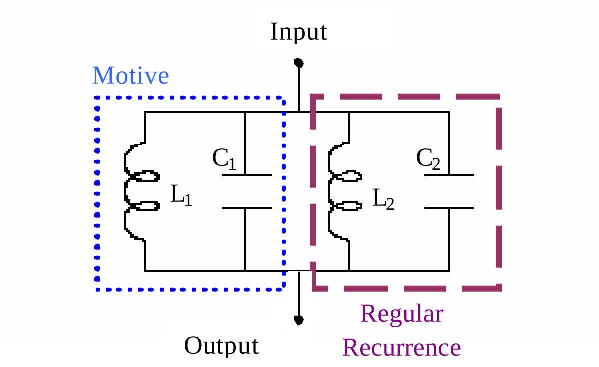

Figure 4. Equivalent circuit model

This kind of analysis is made when the dimensions of the motive are small than l , so the motive is referred as *micromotive*. In our case, the dimensions are approximately l but we use this procedure to be able to have in account the effects that the periodicity has in the isolated ring.

Our interest is focused on the forbidden band, so we try to adjust the frequency responses in this interval more than in the full one. The election of the equivalent circuit is also based on this characteristic: a stop-band filter has this configuration.

The goodness of the proposed circuit has been proved by the realization of several simulations. As an example, a simulation for ε_r =2.6 is shown in Fig. 5.

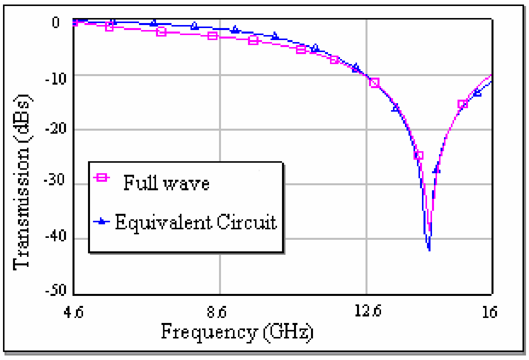

Figure 5. Comparison between full-wave simulation and equivalent circuit for ε_r = 2.6

.

5. WORK METHODOLOGY

Analysis procedure (Fig. 6) is carried out in two steps. First of all, reflection and transmission coefficients (frequency response of the FSS) are calculated by performing full wave domain simulations. Secondly, this frequency response is approximated by adjusting the lumped elements of the equivalent-circuit model. By doing this, inductors and capacitors values are properly calculated. Nevertheless, this is a high computational procedure, so in this work a SVR substitutes this analysis step.

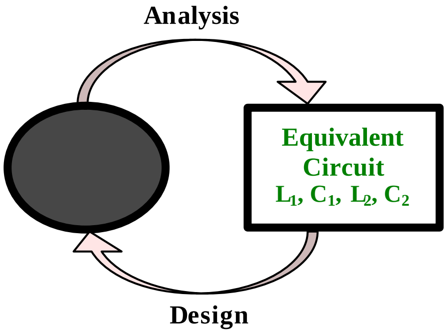

Figure 6. Domain interchange

SVR analysis method is as follows: FSS parameters are introduced into the SVR statistical learning algorithm in order to obtain the element values of the equivalent-circuit model. Training data set is constructed by simulating different FSS configurations (changes in FSS parameters -input samples-) that result in different circuit values (L_1, C_1, L_2 and C2 –output labels -).

As it was explained before, FSS is defined by 6 parameters: D, Dext, Din, t, er, and θ. It must be pointed out that three of them do not need to be included into the SVR training procedure since:

- The effect of both *t* and ε_r on circuit values follows a well known behaviour, which in the case of er can be expressed as:

$$f = \frac{c}{l\sqrt{e_{eff}}} \tag{1}$$

- As it was stated at the beginning of this paper, frequency response of the FSS is independent for θ values below 50º.

6. SUPPORT VECTOR REGRESSION

SVM for regression problems is a sampled-based statistical learning algorithm based on the inductive principle of Structural Risk Minimization. A detailed description of SVR can be found, for instance, at [1].

If the training data is given by $\{(x_1, y_1),\dots,(x_l, y_l)\}$ $\subset \aleph\times\Re^n$, where $\aleph$ denotes the space of the input patterns, the objective of the SVR is to find a function *f(x)* (Fig. 7) which has a maximum deviation ε from the actually obtained targets y_i for all the training data and, at the same time, it is as flat as possible.

When a linear function *f* is used to adjust the data, this function takes the form

$$f(x)=\langle w,x\rangle+b \qquad \text{with} \qquad w\in\aleph, b\in\Re \tag{2}$$

where $\langle\cdot,\cdot\rangle$ denotes the dot product in $\aleph$ and the regression problem (convex optimization problem) can be stated as follow

$$\frac{1}{2}\|w\|^2 \tag{3}$$

$$\text{subject to the constraint} \begin{cases} y_i-\langle w,x_i\rangle-b\le e \\ \langle w,x_i\rangle+b-y_i\le e \end{cases} \tag{4}$$

In the case where the data cannot be adjusted by a linear function input samples are mapped into a higher dimensional space where the regression problem can be solved. When doing this, the approximate function is given by a

$$f(x)=\langle w,f(x)\rangle+b \tag{5}$$

Eq. 6 is a Mercer's kernel, which allows us to calculate the dot product of pairs of vectors transformed by $f(x_i)$ without explicitly knowing the nonlinear mapping (kernel trick).

$$K\left(x_i,x_j\right)=\left(f(x_i)\cdot f\left(x_j\right)\right) \tag{6}$$

Two often used kernels are the linear, given by

$$K\left(x_i,x_j\right)=\left(x_i\cdot x_j\right) \tag{7}$$

and the Gaussian, given by

$$K\left(x_i,x_j\right)=\exp\left\{-\frac{\left\|x_i-x_j\right\|^2}{2s^2}\right\} \tag{8}$$

In the work presented here the kernel type is Gaussian.

When SVR are used, first step is to fix its free parameters (machine configuration). These parameters are ε (maximum deviation) and s (Gaussian Kernel width). Methods such as cross-validation or bootstrap resampling can be used to properly calculate these values. A cross-validation technique (k-fold) [11] has been used in our case.

Sample set is unbalanced so both training and test sets have to be weighted accordingly.

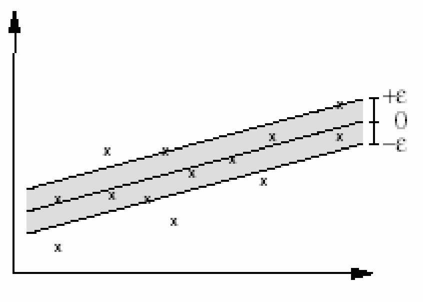

Figure 7. Soft margin loss setting for a linear SVR

It should be noted that after fixing s and ε the SVR is retrained using the entire training set in order to build the FSS analysis procedure model.

7. RESULTS

Results for two different FSS configuration are presented here. Figure 7 shows SVR performance for parameters D_{ext} = 6.1 mm, D_{int} = 4.7 mm and D = 7.24 mm. In the simulation presented in Figure 8 the parameters used were: D_{ext} = 6.1 mm, D_{int} = 4.8 mm and D = 7.24 mm. As can be seen in the previous figures, SVR behaves correctly when comparing to both full wave and equivalent-circuit domains.

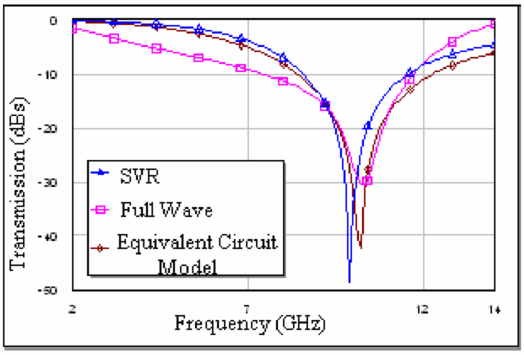

Figure 8. D_{int}=0.47

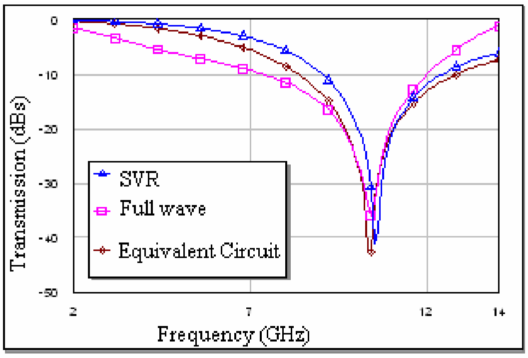

Figure 9. D_{int}=0.48

8. CONCLUSIONS

In this paper we have implemented a statistical learning algorithm for the FSS analysis stage. SVR enables to reduce the computational burden associated with the calculus of the FSS equivalent-circuit parameters in an approximately ratio of 10. Additionally, faster computational time allows us to generate a higher number of lumped element values.

The approach to analysis procedure presented here is the first step to the final objective which would be application of the SVR in the design process of the FSS. Future work will use these previous results to generate a SVR machine that automatically returns physical parameters of the FSS.

9. ACKNOWLEDGMENTS

This work has been supported by MICyT under the project TIC2003-03808.

10. REFERENCES

1. Drucker, H., Burges, C., Kaufman, L., Smola, A., & Vapnik, V. (1997). *Support vector regression machines*. In M. Mozer, M. Jordan, & T. Petsche (Eds.), Advances in Neural Information Processing Systems 9 (pp. 155{161). The MIT Press.
2. Jeremy A. Bossard, Douglas H. Werner, Theresa S. Mayer, and Robert P. Drupp, *A Novel Design Methodology for Reconfigurable Frequency Selective Surfaces Using Genetic Algorithms*, IEEE Transactions on Antennas and Propagation, Vol. 53, No. 4, April 2005. Page(s): 1390- 1400.
3. B. A. Munk, *Frequency Selective Surfaces*: Theory and Design. New York: Wiley, 2000.
4. J. Bardi, R. Remski, D. Perry, and Z. Cendes, *Plane Wave Scattering from Frequency Selective Surfaces by the Finite Element Method*, Magnetics, IEEE Transactions on Magnetics, Volume 38, Issue 2, March 2002 Page(s): 641 – 644.
5. L. Fernández, E. García, D. Castro, and D. Segovia, *Tool to Design Frequency Selective Surfaces Using an Equivalent Circuit Model*, Microwave and Optical Technology Letters, Vol. 47, NO. 5, December 2005. Page(s): 464-467.
6. X. F. Luo, P. T. Teo, A. Qing, and C. K. Lee, *Design of Double-Square-Loop Frequency Selective Surfaces using Differential Evolution Strategy coupled with Equivalent-Circuit Model.* Microwave and Optical Technology Letters / Vol. 44, NO. 2, January 20 2005.
7. Yunus E. Erdemli, Kubilay Sertel, Roland A. Gilbert, Daniel E. Wright, and John L. Volakis, *Frequency-Selective Surfaces to Enhance Performance of Broad-Band Reconfigurable Arrays*, IEEE Transactions on Antennas and Propagation, Vol. 50, NO. 12, December 2002. Page(s):1716 – 1724.
8. Zhi Liang Wang, Kozo Hashimoto, Naoki Shinohara, and Hiroshi Matsumoto, *Frequency-Selective Surface for Microwave Power Transmission*, IEEE Transactions on Microwave Theory and Techniques, VOL. 47, NO. 10, October 1999. Page(s):2039 – 2042.
9. S. B. Savia and E. A. Parker, *Equivalent Circuit Model for Superdense Linear Dipole FSS*. IEE Proc. - Microw. Antennas Propagation, Vol 150, NO. 1, February 2003. Page(s):37 – 42.
10. J. H. Meloling, R. J. Marhefka, *FSS Equivalent Circuit Extraction Techniques*, International Symposium IEEE Antennas and Propation Society, 2003, Volume 2, 22-27 June 2003. Page(s): 793 – 796.
11. Zhang, P., *Model selection via multifold cross-validation*. The Annals of Statistics, 21, pp. 299-313, 1993.